Ian Macdonald

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This note is an extension of remarks made at the meeting and is not intended to be a comprehensive appreciation of Ian's contribution - hopefully others may find the list of publications useful.

Despite his name, Ian (Grant Macdonald) does not claim to be Scottish, his forebears had been in England for some generations. He was born in London in 1928. His father, who himself left school at 14, quite early on noticed that Ian was unusually talented as a mathematician and he was anxious that Ian should get the good education which he had been denied. He managed to get Ian into the prestigious Winchester College which is known as one of the leading Public Schools in England. There, amongst a great deal of other privileges, he received the best possible mathematical education - an education which ensured that each pupil was stretched to his limits and examinations were a mere afterthought. Ian flourished there, but he tells me that just as he was to take scholarship examinations for Trinity College, Cambridge his father asked the housemaster's opinion as to how good Ian really was. The reply came in the end of term report from the senior mathematics master whose verdict was that although Ian was a very good mathematician, he did not think that he was destined to be of 'fellowship' class, that is, of an Oxbridge College. A possible reason for this opinion was the existence of two students who were some five years ahead of Ian at Winchester - namely the distinguished theoretical physicist Freeman Dyson (their paths will cross later) and Sir James Lighthill, who became one of the world's leading applied mathematicians, an FRS at 29, Director of the Royal Aircraft Establishment at Farnborough and ending up as Provost of University College London. So maybe as a result of this the Winchester dons had too exalted opinion of the brain power required of fellows of Oxbridge colleges. For a brilliant account of what was on offer to the talented mathematician at Winchester, read what Freeman Dyson wrote in the special volume written as a tribute to Sir James Lighthill on his premature accidental death in 1998. (Of interest in the context of this meeting is the following remark in a letter from Lighthill to Dyson when both of them were 16 and waiting to take up their places at Trinity College 'What I would like to get is Littlewood's "Theory of Group Characters" (Oxford, 1940) which I saw in Bowes and Bowes and which pleased me greatly but [I am] penniless'.)

Ian proved that his mathematics master had grossly underestimated his ability for he got a Scholarship to enter Trinity College Cambridge to read Mathematics

- furthermore, his subsequent mathematical career proved that he was as good as any in his generation. He took up the scholarship after completing military service and graduated with a BA in 1952. Trinity College is well known for the talent of its undergraduates, but it is hard to believe they could ever have had a better year than Ian's years there. For exact contemporaries with him were, to name only a few, Michael Atiyah, Frank Adams, John Polkinghorne, Ronald Shaw and James Mackay (who after a short period as a lecturer in applied mathematics at St. Andrews changed to law and later became Lord Mackay of Clashfern and Lord Chancellor of Great Britain).

By that time Ian had met up with Greta who is Belgian and they married in 1954 and subsequently had five children - two sons and three daughters (by now, they have fourteen grandchildren). For some reason, Ian decided at that time not to pursue a mathematical career and in 1952 took a position in the civil service as an Assistant Principal and was later promoted to Principal at the Ministry of Supply. But by 1957 he had, fortunately for mathematics, decided that the civil service and administration was not for him and he wanted to return to mathematics. However, he had no formal research experience, no publications but did have a formidable record from his undergraduate studies at Trinity. At that time, Professor Max Newman had built at the University of Manchester an extremely strong Mathematics Department - amongst others on the teaching staff in that period, although not all exact contemporaries, were Frank Adams, Bryan Birch, Paul Cohn, Walter Hayman, Graham Higman, Bernhard Neumann, and possibly more relevant in this context, Sandy Green (all of these were later made FRS's). Newman must have known about Ian's reputation, for he was offered an Assistant Lectureship there in 1957. He may not have had a specific or narrow research field, but this he worked to his advantage for he developed over the years to build an international record as a mathematician over a broad front with acknowledged contributions in algebraic geometry, algebraic groups, number theory and algebraic combinatorics.

Ian's first publication [1] soon appeared in 1958; this contained results which he obtained whilst still in the civil service. It was in algebraic geometry - followed by another [2] in 1960 in a different direction, Jordan algebras. His Assistant Lectureship was only for a three year period and in 1960 he moved to the University of Exeter where Professor David Rees was Head of Department. He stayed there for three years when he was appointed Fellow at Magdalen College, Oxford. This is when he and his family moved into an address which became very familiar to many over the years, 8 Blandford Avenue.

None of Ian's published work over that decade seems to be directly involved with what he ultimately became well known for. In fact, it was not even widely known that he was working during that period on what are now called Hall-Littlewood polynomials. I believe that it was in the late 1960's that I met him for the first time and found that he was preparing a review article for the Deutsche Jahrbuch on the subject. On subsequent occasions I used to ask about progress, but nothing was appearing. Of course, he had been introduced to the subject by his colleague and close friend Sandy Green during his Manchester years and had a thorough knowledge of the whole background (see the Note on Philip Hall). In fact, it was only in this meeting that I discovered that Ian had conjectured the result that Littlewood subsequently proved. In July 1973 at the Mathematics Institute, Warwick Ian presented in a series of 3 lectures on 'Hall polynomials' the material which was developed to be part of his classic book [B4] which appeared in 1979. That book was of course far broader in scope - this was the reason for the 'delay' before its appearance, because it also contained a much needed rigorous introduction to symmetric functions.

In a review written at the time I wrote,

> Although a monograph of modest length, it is encyclopaedic in content, a veritable mine of information. In addition to providing a self-contained and coherent account of well known and classical work, there is a great deal which is original. The book is dotted with gems, both old and new. It is a substantial and valuable volume and will be regarded as the authoritative source which has been long awaited in the subject.

When that was written, I did not foresee the massive impact it would have on the subject and that it would develop to be the classic that it is by now. A number of years later in 1995, a second edition appeared, this is considerably longer, the 180 pages having now evolved into 476 pages, but more about this later for we have moved too far ahead.

For in 1971 [B3] had appeared, the result of a course of lectures at the Ramanujan Institute of Advanced Studies at Madras in 1970 on spherical functions on a group of p-adic type. This publication is still sought after. In 1972, appeared three papers [13], [14] and [15] of great significance. Of these, the most striking is [14], where he derived the Macdonald Identities, which extended certain famous classical power series identities and where he established a connection between a number theoretical observation by the physicist Freeman Dyson mentioned earlier and the theory of simple Lie algebras. These identities aroused widespread interest and had fundamental significance for the representation theory of Kac-Moody algebras. The impact of this paper in this respect is highlighted in the Introduction to Victor G. Kac's book *Infinite Dimensional Lie Algebras - An Introduction* (Birkhauser 1983).

In this remarkable paper [14], Ian obtained a generalization of the Jacobi triple product identity. He showed that there was such an identity for every affine root system, with the classical Jacobi identity corresponding to the affine sl_2 root system. He observed that these identities could be regarded as infinite dimensional analogs of the Weyl denominator formula. However to make the formulas work in the affine setting required him to introduce certain 'mysterious' factors. The attempt to understand these identities provided much of the impetus for the study of the representation theory of Kac-Moody Lie algebras. It is now understood that the mysterious factors correspond to the 'imaginary' roots of the associated Kac-Moody algebra.

There is also an interesting story in this context related by Freeman Dyson in his article 'Missed Opportunities' (Bull. Amer. Math. Soc. 78, (1972), 635 - 652) from which I quote

> This story has a happy ending. Unknown to me the English geometer, Ian Macdonald, had discovered these same formulae as special cases of a much more general theory. In his theory the Lie algebras were incorporated from the beginning, and it was the connection with modular forms which came as a surprise. Anyhow, Macdonald established the connection and so picked up the opportunity which I missed. It happened also that Macdonald was at the Institute for Advanced Study in

> Princeton while we were both working on the problem. Since we had daughters in the same class at school, we saw each other from time to time during his year in Princeton. But since he was a mathematician and I was a physicist, we did not discuss our work. The fact that we had been thinking about the same problem while sitting so close to one another only emerged after he had gone back to Oxford. This was another missed opportunity, but not a tragic one, since Macdonald cleaned up the whole subject very happily without any help from me.

So, the two ex-Winchester schoolboys did meet up, even if that may only have been at a bus stop. It certainly confirmed that his Winchester mathematics master could have been more encouraging. Also, [13] is a beautiful short paper where a classical construction due to Specht is extended in an elegant and suprisingly easy way to Weyl groups in general.

In 1972, Ian was appointed Fielden Professor of Mathematics at the University of Manchester, back where he had started his mathematical employment and replacing his student colleague, Frank Adams, who had returned to Cambridge. He was not there for long however, for in 1976 he was appointed as Professor of Pure Mathematics at Queen Mary and Westfield College, University of London. This meant that he could travel conveniently from 8, Blandford Avenue in Oxford which the family had retained as their 'home' even during their four years in Manchester. His excellence as a researcher was rewarded in 1979 on his election to the Royal Society. Furthermore his contributions as a researcher and his outstanding gifts as a writer and lecturer were recognised through the award of the 1991 Pólya Prize by the London Mathematical Society. In 1998 he was a plenary speaker at the meeting of the International Congress of Mathematics in Berlin.

Also notable is [31] where he formulates some conjectures about the combinatorial properties of root systems - these conjectures were to be the centre of attention in the following twenty years. In 1987/88 he defines a new class of symmetric functions which generalizes the one variable t Hall-Littlewood polynomials to involve two variables q and t, these are now referred to as the Macdonald (q,t)-polynomials; these also generalize Jack polynomials. Also, around the same time, more generally, to a class of polynomials associated with root systems; these are also called Macdonald polynomials. This work, ultimately appeared in [36], [41] and [45], but also in the Second Edition of his Symmetric Functions and Hall Polynomials [B6] in 1995. The impact of these polynomials in mathematics and physics has been enormous and new and deep ideas from various directions were required to eventually prove the conjectures which appeared in this work.

In fact, all of this has now been written up by Ian and appears in his latest volume [B9]. The final writing up of this volume was completed during the six month programme at the Issac Newton Institute, Cambridge in 2001 on Symmetric Functions and Macdonald Polynomials of which he was one of the organisers.

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