Pillay's conjecture for orientable definable groups

Mário Edmundo
Giuseppina Terzo

CMAF
(Centro de Matemática e Aplicações Fundamentais)
Universidade de Lisboa

June 27-30, 2008

LEEDS' 08

Main Result [1]

We assume that $\mathcal{M} = (M, <, \dots)$ is a sufficiently saturated o-minimal structure with definable Skolem functions.

Theorem (E and T)

Let G be a $\mathbb{Z}/q\mathbb{Z}$-orientable, definably connected, definably compact, definable group, where q is some sufficiently large prime number. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Main Result [1]

We assume that $\mathcal{M} = (M, <, \dots)$ is a sufficiently saturated o-minimal structure with definable Skolem functions.

Theorem (E and T)

Let G be a $\mathbb{Z}/q\mathbb{Z}$-orientable, definably connected, definably compact, definable group, where q is some sufficiently large prime number. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Main Result [1]

We assume that $\mathcal{M} = (M, <, \dots)$ is a sufficiently saturated o-minimal structure with definable Skolem functions.

Theorem (E and T)

Let G be a $\mathbb{Z}/q\mathbb{Z}$-orientable, definably connected, definably compact, definable group, where q is some sufficiently large prime number. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Pillay's Conjecture

Our Main Result [1] is a partial solution to:

Conjecture (Pillay)

Let G be a definably connected, definably compact, definable group. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Pillay's Conjecture

Our Main Result [1] is a partial solution to:

Conjecture (Pillay)

Let G be a definably connected, definably compact, definable group. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Pillay's Conjecture

Our Main Result [1] is a partial solution to:

Conjecture (Pillay)

Let G be a definably connected, definably compact, definable group. Then there exists a smallest type definable normal subgroup G^{00} of G of bounded index such that G/G^{00} with the logic topology is a connected, compact, Lie group. Moreover, the following hold:

1. If G is abelian then G^{00} is divisible and torsion free;
2. $\dim G = \dim G/G^{00}$.

Known Cases

1. Field case: [Hrushovski, Peterzil and Pillay]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou and Starchenko]

In particular the conjecture is true in any o-minimal expansion of a group.

Known Cases

1. Field case: [Hrushovski, Peterzil and Pillay]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou and Starchenko]

In particular the conjecture is true in any o-minimal expansion of a group.

Known Cases

1. Field case: [Hrushovski, Peterzil and Pillay]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou and Starchenko]

In particular the conjecture is true in any o-minimal expansion of a group.

Known Cases

1. Field case: [Hrushovski, Peterzil and Pillay]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou and Starchenko]

In particular the conjecture is true in any o-minimal expansion of a group.

Known Cases

1. Field case: [Hrushovski, Peterzil and Pillay]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou and Starchenko]

In particular the conjecture is true in any o-minimal expansion of a group.

Main Result [2]

Theorem (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Then every definably connected, definably compact definable group is k-orientable for every field k.

In particular we obtain a new and uniform proof of Pillay's conjecture in o-minimal expansions of groups.

Orientability of definable groups was already known in the field case: [E], [Berarducci and Otero]. It is still open in the linear case.

Main Result [2]

Theorem (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Then every definably connected, definably compact definable group is k-orientable for every field k.

In particular we obtain a new and uniform proof of Pillay's conjecture in o-minimal expansions of groups.

Orientability of definable groups was already known in the field case: [E], [Berarducci and Otero]. It is still open in the linear case.

Main Result [2]

Theorem (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Then every definably connected, definably compact definable group is k-orientable for every field k.

In particular we obtain a new and uniform proof of Pillay's conjecture in o-minimal expansions of groups.

Orientability of definable groups was already known in the field case: [E], [Berarducci and Otero]. It is still open in the linear case.

Main Result [2]

Theorem (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Then every definably connected, definably compact definable group is k-orientable for every field k.

In particular we obtain a new and uniform proof of Pillay's conjecture in o-minimal expansions of groups.

Orientability of definable groups was already known in the field case: [E], [Berarducci and Otero]. It is still open in the linear case.

Main Result [2]

Theorem (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Then every definably connected, definably compact definable group is k-orientable for every field k.

In particular we obtain a new and uniform proof of Pillay's conjecture in o-minimal expansions of groups.

Orientability of definable groups was already known in the field case: [E], [Berarducci and Otero]. It is still open in the linear case.

Main Result [3]

Theorem (E and T)

Let G be a $\mathbb{Z}/q\mathbb{Z}$-orientable, definably connected, definably compact, definable abelian group, where q is some sufficiently large prime number. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

Main Result [3]

Theorem (E and T)

Let G be a $\mathbb{Z}/q\mathbb{Z}$-orientable, definably connected, definably compact, definable abelian group, where q is some sufficiently large prime number. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Corollary of Main Result [3]

Corollary (E and T)

Suppose that $\mathcal{M}$ is a non linear o-minimal expansion of an ordered group. Let G be a definably connected, definably compact, definable abelian group. Then the subgroup of m-torsion points of G is

$$G[m] \simeq (\mathbb{Z}/m\mathbb{Z})^{\dim G}.$$

This result was already known:

1. Field case: [E and Otero]
2. Semi-bounded case: [Peterzil]
3. Linear case: [Eleftheriou, Starchenko]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

Peterzil - Steinhorn Conjecture

Main Result [3] is a strong solution of:

Conjecture (Peterzil - Steinhorn)

Let G be a definably connected, definably compact, definable abelian group. Then G has a non trivial torsion point.

Other solutions of this were already know in the field case:

- [E]
- [Berarducci and Otero]
- [Berarducci and Otero + Peterzil and Starchenko]
- [Hrushovski, Peterzil and Pillay]

First Results on Pillay's Conjecture

- Statement of the Conjecture
  [Pillay]
- Existence of G^{00}
  [Berarducci, Otero, Peterzil and Pillay], [Shelah]
- Divisibility of G^{00} in the abelian case
  [Berarducci, Otero, Peterzil and Pillay]

First Results on Pillay's Conjecture

- Statement of the Conjecture
  [Pillay]
- Existence of G^{00}
  [Berarducci, Otero, Peterzil and Pillay], [Shelah]
- Divisibility of G^{00} in the abelian case
  [Berarducci, Otero, Peterzil and Pillay]

First Results on Pillay's Conjecture

- Statement of the Conjecture
  [Pillay]

- Existence of G^{00}
  [Berarducci, Otero, Peterzil and Pillay], [Shelah]

- Divisibility of G^{00} in the abelian case
  [Berarducci, Otero, Peterzil and Pillay]

First Results on Pillay's Conjecture

- Statement of the Conjecture
  [Pillay]
- Existence of G^{00}
  [Berarducci, Otero, Peterzil and Pillay], [Shelah]
- Divisibility of G^{00} in the abelian case
  [Berarducci, Otero, Peterzil and Pillay]

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Main Tools for Pillay's Conjecture

- Computation of torsion subgroups
  [E and Otero], [Peterzil], [Eleftheriou and Starchenko], [E and T]

- Theory of generic subsets
  [Peterzil and Pillay]
  Reduces the proof of the abelian case to computation of torsion subgroups. Under assumption G affine. Assumption removed: Linear case: [Eleftheriou]. Semi-bounded case: [Peterzil]. General case: [E and T].

- Model theory of definable amenable groups
  [Hrushovski, Peterzil and Pillay]
  Reduces the proof to the abelian case

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Reduction to Computation of Torsions Subgroups

- Theory of generic subsets
- G^{00} is torsion free. But also divisible.
- $G/G^{00}[k] \simeq G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^{\dim G}$

  Therefore $\dim G/G^{00} = \dim G$.

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

Main Tools for Torsion Subgroups

- O-minimal fundamental groups.
  Field case: [E], [Beraducci and Otero], [E and Otero].
  Group case: [Eleftheriou and Starchenko], [E and Eleftheriou].
  General case: [E, Peatfield and Jones].

- O-minimal cohomology.
  Field case: [Woerheide], [E], [E and Woerheide], [Berarducci and Otero], [E and Otero].
  General case: [E, Peatfield and Jones], [E, Peatfield], [Berarducci], [Berarducci and Fornasiero], [E], [E and T].

O-minimal Fundamental Groups

Definition

In o-minimal expansions of groups the o-minimal fundamental group of G,

$$\pi_1(G)$$

is defined using definable paths and definable homotopies.

Definition (E, Peatfield and Jones)

In arbitrary o-minimal structures the o-minimal fundamental group of G,

$$\eta(G)$$

is defined by

$$\eta(G) = \varprojlim_{K \longrightarrow H \in \mathrm{MorCov}^0_{\mathrm{def}}(G)} \ker(K \longrightarrow H).$$

O-minimal Fundamental Groups

Definition

In o-minimal expansions of groups the o-minimal fundamental group of G,

$$\pi_1(G)$$

is defined using definable paths and definable homotopies.

Definition (E, Peatfield and Jones)

In arbitrary o-minimal structures the o-minimal fundamental group of G,

$$\eta(G)$$

is defined by

$$\eta(G) = \varprojlim_{K \longrightarrow H \in \mathrm{MorCov}^0_{\mathrm{def}}(G)} \ker(K \longrightarrow H).$$

O-minimal Fundamental Groups

Definition

In o-minimal expansions of groups the o-minimal fundamental group of G,

$$\pi_1(G)$$

is defined using definable paths and definable homotopies.

Definition (E, Peatfield and Jones)

In arbitrary o-minimal structures the o-minimal fundamental group of G,

$$\eta(G)$$

is defined by

$$\eta(G) = \varprojlim_{K \longrightarrow H \in \mathrm{MorCov}^0_{\mathrm{def}}(G)} \ker(K \longrightarrow H).$$

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups we have

$$\eta(G) = \widehat{\pi_1(G)}$$

(profinite completion.)

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups we have

$$\eta(G) = \widehat{\pi_1(G)}$$

(profinite completion.)

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].
Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \Pi_{p \text{ prime}} \Pi_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].
Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \prod_{p \text{ prime}} \prod_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].

Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \Pi_{p \text{ prime}} \Pi_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].
Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \prod_{p \text{ prime}} \prod_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].
Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \Pi_{p \text{ prime}} \Pi_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Fundamental Groups

Theorem (E and Eleftheriou)

In o-minimal expansions of groups, if G is abelian we have:

1. $\pi_1(G) \simeq \mathbb{Z}^s$.
2. $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$.

Field case: [E and Otero].

Linear case: with $s = \dim G$ [Eleftheriou and Starchenko].

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, if G is abelian we have:

1. $\eta(G) \simeq \Pi_{p \text{ prime}} \Pi_{s(p)} \mathbb{Z}_p$.
2. $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$ (ℓ prime).

So we need to show that $s = \dim G$ and resp. $s(\ell) = \dim G$.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1, \ldots, \omega_t]_{\mathbb{Q}}$.
2. each $\mathrm{deg}\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \mathrm{deg}\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1,\ldots,\omega_t]_{\mathbb{Q}}$.
2. each $\deg\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \deg\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \deg\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1, \ldots, \omega_t]_{\mathbb{Q}}$.
2. each $\mathrm{deg}\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \mathrm{deg}\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1, \ldots, \omega_t]_{\mathbb{Q}}$.
2. each $\deg\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \deg\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \deg\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1, \ldots, \omega_t]_{\mathbb{Q}}$.
2. each $\mathrm{deg}\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \mathrm{deg}\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have:

1. $H^*(G;\mathbb{Q}) \simeq \bigwedge[\omega_1, \ldots, \omega_t]_{\mathbb{Q}}$.
2. each $\mathrm{deg}\omega_i$ is odd and $t \leq \sum_{i=1}^{t} \mathrm{deg}\omega_i \leq \dim G$.
3. since G is orientable, $\sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$.

Here we work with o-minimal singular cohomology.

Requires:

Künneth formula for o-minimal singular cohomology.

Poincaré duality for o-minimal singular cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G; \mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1, \ldots, \omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\deg \omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \deg \omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \deg \omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G; \mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1, \ldots, \omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\deg \omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \deg \omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \deg \omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G;\mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1,\ldots,\omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\deg\omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \deg\omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \deg\omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G; \mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1, \ldots, \omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\deg \omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \deg \omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \deg \omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G; \mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1, \ldots, \omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\mathrm{deg}\omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \mathrm{deg}\omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \mathrm{deg}\omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

O-minimal Cohomology: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures with definable Skolem functions, if $q > \dim G + 1$, q prime, we have:

1. $H^*(G; \mathbb{Z}/q\mathbb{Z}) \simeq \bigwedge[\omega_1, \ldots, \omega_{t(q)}]_{\mathbb{Z}/q\mathbb{Z}}$.
2. each $\deg \omega_i$ is odd and $t(q) \leq \sum_{i=1}^{t(q)} \deg \omega_i \leq \dim G$.
3. if G is $\mathbb{Z}/q\mathbb{Z}$-orientable, $\sum_{i=1}^{t(q)} \deg \omega_i = \dim G$.

Here we work with o-minimal sheaf cohomology.

Requires:

Künneth formula for o-minimal sheaf cohomology.

Poincaré duality for o-minimal sheaf cohomology.

Degree Theory: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields, if G is abelian and

$$p_k : G \longrightarrow G : x \mapsto kx,$$

then

$$|G[k]| = \deg p_k = k^t.$$

Hence $s = t \leq \dim G$. (Since $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$).

Degree Theory: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields, if G is abelian and

$$p_k : G \longrightarrow G : x \mapsto kx,$$

then

$$|G[k]| = \deg p_k = k^t.$$

Hence $s = t \leq \dim G$. (Since $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$).

Degree Theory: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields, if G is abelian and

$$p_k : G \longrightarrow G : x \mapsto kx,$$

then

$$|G[k]| = \mathrm{deg} p_k = k^t.$$

Hence $s = t \leq \dim G$. (Since $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$).

Degree Theory: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields, if G is abelian and

$$p_k : G \longrightarrow G : x \mapsto kx,$$

then

$$|G[k]| = \mathrm{deg} p_k = k^t.$$

Hence $s = t \leq \dim G$. (Since $G[k] \simeq (\mathbb{Z}/k\mathbb{Z})^s$).

Degree Theory: General Case

Theorem (E and T)

In arbitrary o-minimal structures with definable Skolem functions, if G is abelian,

$$p_\ell : G \longrightarrow G : x \mapsto \ell x$$

and $\ell > \dim G + 1$, $q > \ell^{\dim G}$ are primes such that G is $\mathbb{Z}/q\mathbb{Z}$-orientable, then

$$|G[\ell]| = \deg p_\ell = \ell^{t(q)}.$$

Hence $s(\ell) = t(q) \leq \dim G$. (Since $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$).

Degree Theory: General Case

Theorem (E and T)

In arbitrary o-minimal structures with definable Skolem functions, if G is abelian,

$$p_\ell : G \longrightarrow G : x \mapsto \ell x$$

and $\ell > \dim G + 1$, $q > \ell^{\dim G}$ are primes such that G is $\mathbb{Z}/q\mathbb{Z}$-orientable, then

$$|G[\ell]| = \deg p_\ell = \ell^{t(q)}.$$

Hence $s(\ell) = t(q) \leq \dim G$. (Since $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$).

Degree Theory: General Case

Theorem (E and T)

In arbitrary o-minimal structures with definable Skolem functions, if G is abelian,

$$p_\ell : G \longrightarrow G : x \mapsto \ell x$$

and $\ell > \dim G + 1$, $q > \ell^{\dim G}$ are primes such that G is $\mathbb{Z}/q\mathbb{Z}$-orientable, then

$$|G[\ell]| = \deg p_\ell = \ell^{t(q)}.$$

Hence $s(\ell) = t(q) \leq \dim G$. (Since $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$).

Degree Theory: General Case

Theorem (E and T)

In arbitrary o-minimal structures with definable Skolem functions, if G is abelian,

$$p_\ell : G \longrightarrow G : x \mapsto \ell x$$

and $\ell > \dim G + 1$, $q > \ell^{\dim G}$ are primes such that G is $\mathbb{Z}/q\mathbb{Z}$-orientable, then

$$|G[\ell]| = \deg p_\ell = \ell^{t(q)}.$$

Hence $s(\ell) = t(q) \leq \dim G$. (Since $G[\ell] \simeq (\mathbb{Z}/\ell\mathbb{Z})^{s(\ell)}$).

Hurewicz Theorem: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have

$$\pi_1(G) \simeq H_1(G; \mathbb{Z}).$$

Hence for G abelian (and orientable) we have

$$s = t = \dim G.$$

(Since $H^*(G;\mathbb{Q}) \simeq \mathrm{Hom}(H_*(G;\mathbb{Z}), \mathbb{Q})$, each $\mathrm{deg}\omega_i = 1$ and so $t = \sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$).

Hurewicz Theorem: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have

$$\pi_1(G) \simeq H_1(G;\mathbb{Z}).$$

Hence for G abelian (and orientable) we have

$$s = t = \dim G.$$

(Since $H^*(G;\mathbb{Q}) \simeq \mathrm{Hom}(H_*(G;\mathbb{Z}),\mathbb{Q})$, each $\mathrm{deg}\omega_i = 1$ and so $t = \sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$).

Hurewicz Theorem: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have

$$\pi_1(G) \simeq H_1(G; \mathbb{Z}).$$

Hence for G abelian (and orientable) we have

$$s = t = \dim G.$$

(Since $H^*(G; \mathbb{Q}) \simeq \mathrm{Hom}(H_*(G; \mathbb{Z}), \mathbb{Q})$, each $\deg \omega_i = 1$ and so $t = \sum_{i=1}^{t} \deg \omega_i = \dim G$).

Hurewicz Theorem: Field Case

Theorem (E and Otero)

In o-minimal expansions of fields we have

$$\pi_1(G) \simeq H_1(G; \mathbb{Z}).$$

Hence for G abelian (and orientable) we have

$$s = t = \dim G.$$

(Since $H^*(G; \mathbb{Q}) \simeq \mathrm{Hom}(H_*(G; \mathbb{Z}), \mathbb{Q})$, each $\mathrm{deg}\omega_i = 1$ and so $t = \sum_{i=1}^{t} \mathrm{deg}\omega_i = \dim G$).

Hurewicz Theorem: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, for every finite abelian group Z, we have a canonical homomorphism

$$\mathrm{Hom}_{\mathrm{cont}}(\eta(G), Z) \longrightarrow H^1(G; Z)$$

which sends surjections to non zero elements.

Hence for G abelian (and $\mathbb{Z}/q\mathbb{Z}$-orientable) we have

$$s(\ell) = t(q) = \dim G.$$

(Since each $\deg\omega_i = 1$ and so $t(q) = \sum_{i=1}^{t(q)} \deg\omega_i = \dim G$).

THE END

Hurewicz Theorem: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, for every finite abelian group Z, we have a canonical homomorphism

$$\mathrm{Hom}_{\mathrm{cont}}(\eta(G), Z) \longrightarrow H^1(G; Z)$$

which sends surjections to non zero elements.

Hence for G abelian (and $\mathbb{Z}/q\mathbb{Z}$-orientable) we have

$$s(\ell) = t(q) = \dim G.$$

(Since each $\deg \omega_i = 1$ and so $t(q) = \sum_{i=1}^{t(q)} \deg \omega_i = \dim G$).

THE END

Hurewicz Theorem: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, for every finite abelian group Z, we have a canonical homomorphism

$$\mathrm{Hom}_{\mathrm{cont}}(\eta(G), Z) \longrightarrow H^1(G; Z)$$

which sends surjections to non zero elements.

Hence for G abelian (and $\mathbb{Z}/q\mathbb{Z}$-orientable) we have

$$s(\ell) = t(q) = \dim G.$$

(Since each $\deg \omega_i = 1$ and so $t(q) = \sum_{i=1}^{t(q)} \deg \omega_i = \dim G$).

THE END

Hurewicz Theorem: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, for every finite abelian group Z, we have a canonical homomorphism

$$\mathrm{Hom}_{\mathrm{cont}}(\eta(G), Z) \longrightarrow H^1(G; Z)$$

which sends surjections to non zero elements.

Hence for G abelian (and $\mathbb{Z}/q\mathbb{Z}$-orientable) we have

$$s(\ell) = t(q) = \dim G.$$

(Since each $\deg \omega_i = 1$ and so $t(q) = \sum_{i=1}^{t(q)} \deg \omega_i = \dim G$).

THE END

Hurewicz Theorem: General Case

Theorem (E, Peatfield and Jones)

In arbitrary o-minimal structures, for every finite abelian group Z, we have a canonical homomorphism

$$\mathrm{Hom}_{\mathrm{cont}}(\eta(G), Z) \longrightarrow H^1(G; Z)$$

which sends surjections to non zero elements.

Hence for G abelian (and $\mathbb{Z}/q\mathbb{Z}$-orientable) we have

$$s(\ell) = t(q) = \dim G.$$

(Since each $\mathrm{deg}\omega_i = 1$ and so $t(q) = \sum_{i=1}^{t(q)} \mathrm{deg}\omega_i = \dim G$).

THE END